

First-Order Resolution

Example from the book (Pg. 298-299)

Everyone who loves all animals is loved by somebody, but anybody who kills an animal is loved by nobody. Jack loves all animals. Either Jack or Curiosity killed the cat, who is named Tuna. Did Curiosity kill the cat?

I. Translate each sentence into FOL and convert to CNF

1. Everyone who loves all animals is loved by somebody.

$$\forall x \{ \forall y [\text{Animal}(y) \Rightarrow \text{Loves}(x, y)] \Rightarrow \exists y [\text{Loves}(y, x)] \}$$

$$\forall x \{ \exists y [\text{Animal}(y) \wedge \neg \text{Loves}(x, y)] \vee \exists z [\text{Loves}(z, x)] \}$$

$$\{ [\text{Animal}(F(x)) \wedge \neg \text{Loves}(x, F(x))] \vee [\text{Loves}(G(x), x)] \}$$

$$[\text{Animal}(F(x)) \vee \text{Loves}(G(x), x)] \wedge [\text{Loves}(G(x), x) \vee \neg \text{Loves}(x, F(x))]$$

2. Anybody who kills an animal is loved by nobody.

$$\forall x \{ \exists y [\text{Animal}(y) \wedge \text{Kills}(x, y)] \Rightarrow \forall y [\neg \text{Loves}(y, x)] \}$$

$$\forall x \{ \forall y [\neg \text{Animal}(y) \vee \neg \text{Kills}(x, y)] \vee \forall z [\neg \text{Loves}(z, x)] \}$$

$$[\neg \text{Animal}(y) \vee \neg \text{Kills}(x, y) \vee \neg \text{Loves}(z, x)]$$

3. Jack loves all animals.

$$\forall x [\text{Animal}(x) \Rightarrow \text{Loves}(\text{Jack}, x)]$$

$$\neg \text{Animal}(x) \vee \text{Loves}(\text{Jack}, x)$$

4. Either Jack or Curiosity killed the cat, who is named Tuna.

$$[\text{Kills}(\text{Jack}, \text{Tuna}) \vee \text{Kills}(\text{Curiosity}, \text{Tuna})]$$

$$[\text{Cat}(\text{Tuna})]$$

$$\forall x [\text{Cat}(x) \Rightarrow \text{Animal}(x)]$$

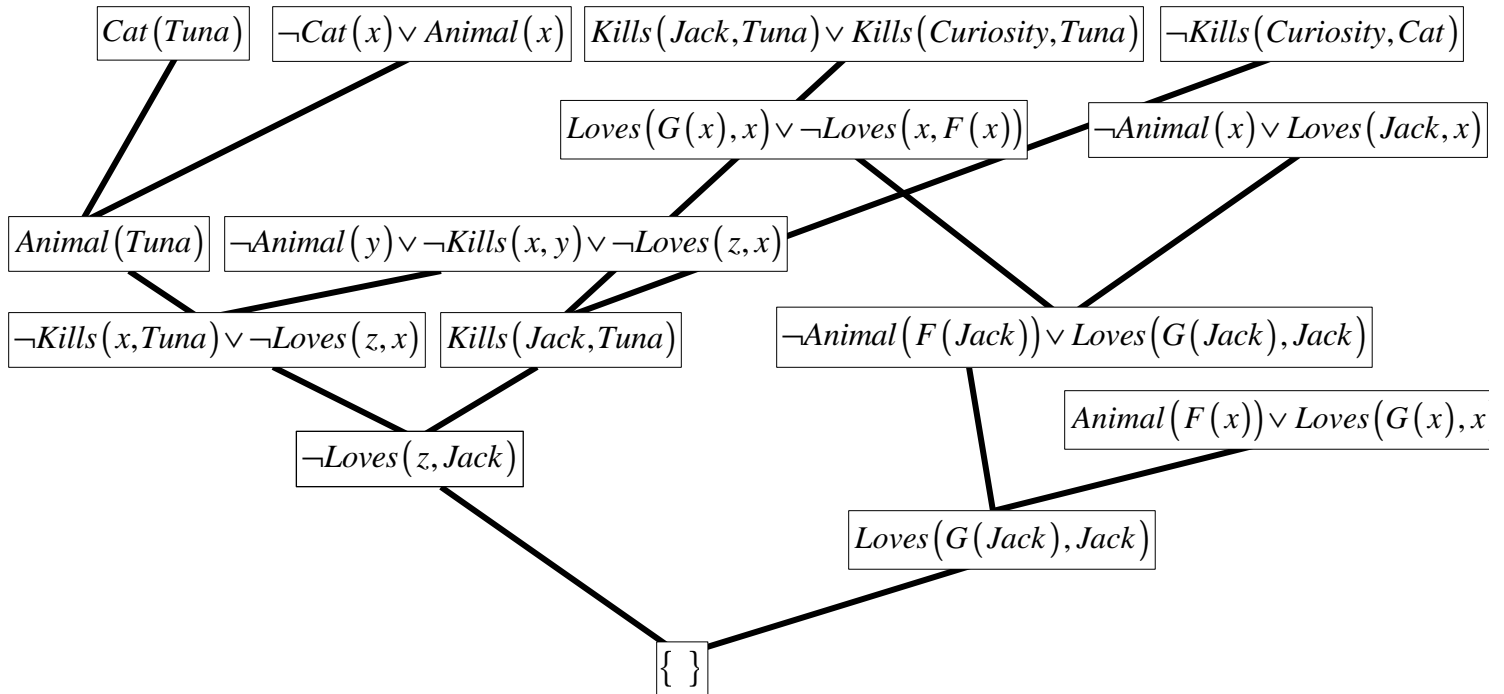
$$[\neg \text{Cat}(x) \vee \text{Animal}(x)]$$

5. Did Curiosity kill the cat?

$$[\text{Kills}(\text{Curiosity}, \text{Cat})]$$

II. Proof by contradiction through resolution

1. Negate the query. $\lceil \neg \text{Kills}(\text{Curiosity}, \text{Cat}) \rceil$
2. Begin resolution – can we derive $\{\}$?



III. Look at the resulting proof

Suppose Curiosity did not kill Tuna. Since either Tuna or Curiosity killed Tuna, Jack did. Moreover, Tuna is a cat, which is an animal, so Tuna is an animal. Since anybody who kills an animal is loved by nobody, nobody loves Jack. However, Jack loves all animals, so somebody loves him, but that is a contradiction. Thus, Curiosity killed Tuna.